

**Mathematics Specialist Year 11**

Student name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Teacher name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date: Friday 23rd July 2021

**Task type: Response**

**Time allowed: 45 minutes**

**Number of questions: 6**

**Materials required:** Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on two unfolded sheets of   
A4 paper, and up to three calculators approved for use in the WACE examinations



**Marks available: 40 marks**

**Task weighting: 10%**

**Formula sheet provided: Yes**

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

1. [7 marks]

Use mathematical induction to prove that

for all positive integers .

|  |
| --- |
| **Solution** |
| Let denote the proposition ‘’ for all positive integers .  With ,  Hence LHS=RHS, and so is true.  Now assume that is true for some positive integer . Then  Now  Hence is true.  We have shown that is true, and that if is true for some positive integer then is also true. Hence, by the principle of mathematical induction, is true for all positive integers . |
| **Specific behaviours** |
| 🗸 proves by **evaluating LHS and RHS separately**  🗸 assumes is true  🗸 writes LHS of using RHS of  🗸 simplifies expression algebraically to one fraction  🗸 writes numerator with a factor of  🗸 obtains expression for RHS of **written in terms of**  🗸 writes conclusion for whole proof (accept just the second sentence without the first) |

1. [2 marks]

A question in a Specialist exam paper asked students to prove the following statement:

‘ is odd if and only if is odd (where is an integer)’.

One student wrote the answer below. Explain clearly why they should **not** receive full marks for this answer.

*Proof:*

*We prove the contrapositive. Assume that is an even integer. Then for some integer . Now*

*which is even since is an integer. Hence if is even then is even, which implies that is odd if and only if is odd.*

|  |
| --- |
| **Solution** |
| The student has proved only the statement ‘if is odd then is odd’. However, since the original statement involves the phrase ‘if and only if’, it is also necessary to prove the statement ‘if is odd then is odd’. |
| **Specific behaviours** |
| 🗸 Notes that statement involves ‘if and only if’, or describes as an equivalence statement  🗸 Explains that the student should also have proved that ‘if is odd then is odd’, or refers to the ‘backward direction’ |

1. [9 = 3+3+3 marks]

Write whether each of the following statements is true or false, and prove or disprove it accordingly.

1. For all positive real numbers

|  |
| --- |
| **Solution** |
| The statement is **false**, and is disproved with the following counterexample:  Let . Then LHS = and RHS = , meaning that in this case.  Hence the statement is false. |
| **Specific behaviours** |
| 🗸 states false  🗸 states counterexample with a particular value of  🗸 shows that for that value of , .  [Alternatively give 2nd and 3rd marks if successfully argues false for any value of with .] |

1. There exist distinct prime numbers and such that .

|  |
| --- |
| **Solution** |
| The statement is **true**, and is proved with the following example:  Let and . Then . |
| **Specific behaviours** |
| 🗸 states true  🗸🗸 states example with values of and such that |

1. There exist distinct prime numbers and such that .

|  |
| --- |
| **Solution** |
| The statement is **false**.  Let and be distinct prime numbers. Then  Since and are distinct primes, and , and so .  Hence there do not exist distinct prime numbers and with . |
| **Specific behaviours** |
| 🗸 states false  🗸 factorises using difference of squares  🗸 argues that cannot equal . |

1. [6 marks]

Find the values of and in each of the following:

1. Diagram

   Description automatically generated and all lie on the circle with centre :

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸🗸🗸 1 mark per correct value |

1. Diagram

   Description automatically generated is tangent to the circle with centre .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸🗸🗸 1 mark per correct value |

1. [5 marks]

is a quadrilateral such that each of the four sides is tangent to the same circle, at the points and , as illustrated below. If , and , find the length .

Diagram

Description automatically generated

|  |
| --- |
| **Solution** |
| Since the sides are tangent to the circle, we may write:  , , and .  Thus  (1)  (2)  (3)  Adding equations (1) and (3) gives  and subtracting equation (2) gives  Hence . |
| **Specific behaviours** |
| 🗸 uses theorem for tangent segments from the same point  🗸 identifies segments of equal lengths  🗸 sets up equations for side lengths using sums of segment lengths  🗸 solves set of equations for  🗸 states correct value  [Accept alternative methods.] |

1. [11 = 3+4+4 marks]

Solve each of the following trigonometric equations for in the stated domain.

**Show all working to support your answers.**

1. for

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 isolates  🗸 states at least one correct solution  🗸 states two correct solutions  [No marks for answers only] |

1. for

|  |
| --- |
| **Solution** |
| Hence  With , or  With , or  Hence or |
| **Specific behaviours** |
| 🗸 isolates  🗸 states at least one correct solution for  🗸 states at least one correct solution or for  🗸 states all three correct solutions for  [No marks for answers only] |

1. for

|  |
| --- |
| **Solution** |
| Letting and we obtain:  and |
| **Specific behaviours** |
| 🗸 isolates  🗸 obtains as a solution for  🗸 obtains as a solution for  🗸 states all five correct solutions for  [No marks for answers only] |